Noise Reduction in Satellite Imagery Using Singular Value Decomposition

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***Abstract*—Satellite imagery processing plays an important role in various applications. However, the quality of satellite imagery is often affected by noise. One effective method for reducing noise in satellite images is the Singular Value Decomposition (SVD) approach. This method utilizes matrix decomposition to separate the main components of the image, allowing the noise to be removed without losing important details. This research paper examines the implementation of the SVD method in the noise reduction process of satellite imagery and analyzes its effectiveness. The results prove that the method can be used for noise reduction, but still requires further modification to improve its effectiveness.**

Keywords—Digital Image Processing, Noise Reduction, Satellite Image, Singular Value Decomposition

I. INTRODUCTION

Satellite imagery has become a crucial data source in various fields, such as environmental monitoring, regional planning, natural resource management, and disaster mitigation. The data obtained from satellite images is invaluable for spatial and temporal analysis, enabling researchers, policymakers, and organizations to make informed decisions based on real-time or historical data. The ability to observe large-scale phenomena and monitor changes over time is critical in addressing a variety of global challenges.

However, despite the significant benefits that satellite imagery offers, the quality of these images is often compromised due to the presence of noise. Noise can emerge during several stages of image acquisition, transmission, or data storage, and it can manifest in various forms, such as pixel irregularities, blurring, or interference. These imperfections can detract from the quality of the image and significantly reduce the accuracy of analysis, especially in sensitive tasks that demand high precision, such as transformation detection, land use classification, and natural resource management. The presence of noise in satellite images can lead to unreliable conclusions, making it difficult to extract meaningful insights from the data.

To overcome this problem, noise reduction methods that can eliminate noise without removing important details from the images are needed. Various methods have been developed to tackle this problem, one of which is the Singular Value Decomposition (SVD)-based approach. SVD is a matrix decomposition technique that allows the separation of the main components in an image from the noise components, resulting in a cleaner and more informative image.

This research aims to examine the application of the SVD method in the process of noise reduction in satellite images. This research will discuss the basic principles of SVD in noise reduction, implement it in program code, and analyse its effectiveness through experimental tests. With this approach, it is expected that noise can be minimized without losing important information, thus improving the quality of the images for further analysis.

II. THEORITICAL BASIS

A. Eigenvalues and Eigenvectors

If A is an n x n matrix, then a non-zero vector $\mathbf{x} \in R^n$ is called an eigenvector of A if A$\mathbf{x}$ is equal to the product of a scalar λ with $\mathbf{x}$, i.e.

$$A\mathbf{x} = \lambda\mathbf{x}$$

The scalar λ is called the eigenvalue of A, and x is called the eigenvector that corresponds to λ. The eigenvectors and eigenvalues of matrix A are calculated as follows:

$$A\mathbf{x} = \lambda\mathbf{x}$$
$$IA\mathbf{x} = \lambda I\mathbf{x}$$
$$A\mathbf{x} = \lambda I\mathbf{x}$$
$$(\lambda I - A)\mathbf{x} = 0$$

$\mathbf{x} = 0$ is a trivial solution of $(\lambda I - A)\mathbf{x} = 0$

For $(\lambda I - A)\mathbf{x} = 0$ to have a non-zero solution, it must be that

$$\det(\lambda I - A) = 0$$

A square matrix A is said to be diagonalizable if it is similar to a diagonal matrix, meaning there exists a matrix P such that $P^{-1}AP$ is a diagonal matrix. Let D be the diagonal matrix, then:
$A = PDP^{-1} \rightarrow D = P^{-1}AP$

B. Singular Value Decomposition (SVD)

SVD is a method for factorizing a matrix A of size m × n into three matrices U, Σ, and V such that:

$$A = U\Sigma V^T$$

where

- U is an orthogonal matrix of size m × m,
- V is an orthogonal matrix of size n × n,
- Σ is a matrix of size m × n whose diagonal elements are the singular values of matrix A, and all other elements are 0.

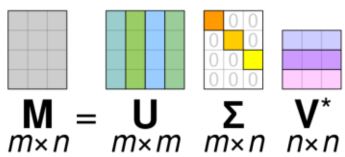

Figure 1. SVD illustration
Source:

https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-21-Singular-value-decomposition-Bagian1-2023.pdf

If A is an m × n matrix with rank k, then A can be factorized as:

$$A = U\Sigma V^T = [\mathbf{u}_1 \ \mathbf{u}_2 \ \cdots \ \mathbf{u}_k | \mathbf{u}_{k+1} \ \cdots \ \mathbf{u}_m] \left[\begin{array}{cccc|c} \sigma_1 & 0 & \cdots & 0 & \\ 0 & \sigma_2 & \cdots & 0 & 0_{k\times(n-k)} \\ \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & \cdots & \sigma_k & 0_{(m-k)\times(n-k)} \\ & 0_{(m-k)\times k} & & & \end{array}\right] \left[\begin{array}{c} \mathbf{v}_1^T \\ \mathbf{v}_2^T \\ \vdots \\ \mathbf{v}_k^T \\ \hline \mathbf{v}_{k+1}^T \\ \vdots \\ \mathbf{v}_n^T \end{array}\right]$$

Figure 2. Matrix A decomposition with SVD
Source:
https://informatika.stei.itb.ac.id/~rinaldi.munir/AljabarGeometri/2023-2024/Algeo-21-Singular-value-decomposition-Bagian1-2023.pdf
ehere

where

- U an m x m matrix, Σ is an m x n matrix, dan V adalah matriks n x n
- The vectors $u_1, u_2, \ldots, u_k$ are called the left singular vectors of matrix A.
- The vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are called the right singular vectors of matrix A
- $\sigma_1 = \sqrt{\lambda_1}, \sigma_2 = \sqrt{\lambda_2}, \ldots, \sigma_k = \sqrt{\lambda_k}$ are singular values of A, and $\lambda_1, \lambda_2, \ldots, \lambda_k$ are eigenvalues of A^TA
- $V = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n]$ orthogonally diagonalizes A^TA
- The columns of V are ordered such that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_k > 0$
- $\mathbf{u}_i = \frac{A\mathbf{v}_i}{||A\mathbf{v}_i||} = \frac{1}{\sigma_i}A\mathbf{v}_i, i = 1, 2, \ldots, k$
- The set $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ is an orthonormal basis for the column space of A (col(A))
- The set $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k, \mathbf{u}_{k+1}, \ldots, \mathbf{u}_m\}$ is an extension of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ to form an orthonormal basis for the vector space R^m

There are two methods for calculating SVD:

1. Method 1:
   a. Determine the right singular vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ corresponding to the eigenvalues of A^TA. Normalize $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ by dividing each vector component by the vector's length. This gives the matrix V. Transpose the matrix V to obtain V^T. The rank of A, denoted Rank(A) = k, is the number of non-zero eigenvalues of A^TA.
   b. Determine the left singular vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k$ using the equation
   $$\boldsymbol{u}_i = \frac{A\boldsymbol{v}_i}{||A\boldsymbol{v}_i||} = \frac{1}{\sigma_i}A\boldsymbol{v}_i, i = 1, 2, \ldots, k$$
   Normalize $\mathbf{u}_1, \mathbf{u}_2, \ldots, u_k$ by dividing each component of the vectors by the vector's length.
   c. If n > k, extend the set $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$ to form an orthonormal basis for R^m.
   d. Construct the matrix Σ of size m × n, with diagonal elements being the non-zero singular values of matrix A, arranged from largest to smallest. The singular values in Σ are the square roots of the non-zero eigenvalues of A^TA.
   e. Thus, $A = U\Sigma V^T$.
2. Method 2:
   a. For the left singular vectors, compute the eigenvalues of AA^T. The rank of A, denoted Rank(A) = k, is the number of non-zero eigenvalues of AA^T.
   b. Determine the eigenvectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m$ corresponding to the eigenvalues of AA^T. Normalize $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m$ by dividing each vector component by the vector's length. This gives the matrix U.
   c. For the right singular vectors, compute the eigenvalues of A^TA and then determine the singular values.
   d. Determine the eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ corresponding to the eigenvalues of A^TA. Normalize $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ by dividing each vector component by the vector's length. This gives the matrix V. Transpose the matrix V to obtain V^T.
   e. Construct the matrix Σ of size m × n, with diagonal elements being the non-zero singular values of matrix A, arranged from largest to smallest. The singular values in Σ are the square roots of the non-zero eigenvalues of A^TA.
   f. Thus, $A = U\Sigma V^T$. → $D = P^{-1}AP$

C. Noise Reduction with SVD

Noise reduction is one of the important fields in digital image processing. Noise in the context of this paper refers to random disturbances or fluctuations that appear in digital images, often seen as fine grains or unwanted spots. The presence of noise can make the photo appear less sharp and cover important details, thus reducing the overall quality of the image. Noise is typically more noticeable in dark areas or when the image is enlarged. Noise can originate from various sources, such as the environment, hardware, or errors in data transmission.

Based on their features and characteristics, noise in images can be classified into several types:

1. Gaussian Noise: Random noise that follows a normal distribution, causing subtle changes in pixel intensity throughout the image.
2. Salt and Pepper Noise: Impulsive noise that appears as white (salt) and black (pepper) pixels randomly scattered in the image.
3. Speckle Noise: Multiplicative noise in the form of spots that arises from the interference of coherent waves, commonly found in wave-based images like ultrasound or radar.

(a) (b) (c)

Figure 3. Noise types: (a) gaussian, (b) speckle, and (c) salt and pepper

Source: https://nana.lecturer.pens.ac.id/PrakCitra09/TeoriReduksiNoise+SNR.pdf

Noise reduction in images is the process of minimizing noise without removing important details in the image, making the image clearer and easier to analyze or process further. The noise reduction process is carried out using filter techniques or mathematical transformations designed to identify and reduce noise. Various methods have been developed to reduce noise, both in the spatial domain and in the transformation domain. Effective noise reduction techniques can improve image quality and preserve relevant image details.

One method that can be used in noise reduction is the application of SVD (Singular Value Decomposition). In image processing, SVD can be used to reduce noise by utilising the fact that noise is typically contained in small singular values. This process is carried out by:

1. Performing SVD decomposition on the noisy image.
2. Filtering out the small singular values, as these are generally the insignificant noise.
3. Reconstructing the image matrix with the larger singular values, which represent the main structure of the image without the noise.

This process is called thresholding of singular values, where singular values below a certain threshold are removed, leaving only the main components. This process will produce a cleaner image by reducing the noise disturbances typically contained in the small singular values.

D. Satellite Imagery

Satellite imagery refers to images captured by sensors mounted on satellites orbiting the Earth. These satellites use a variety of sensors to collect data across different wavelengths, from visible light to infrared, to even microwave radiation. The data obtained from satellite imagery provides valuable insights into a wide range of applications, from environmental monitoring and regional planning to natural resource management and disaster response. By capturing large-scale images of the Earth, satellite imagery allows for the monitoring of changes over time, helping decision-makers track and analyze various phenomena such as deforestation, land use, climate change, and the spread of wildfires.

Satellite images are obtained through remote sensing technology, which enables the collection of data without direct physical contact with the Earth's surface. This technology works by emitting signals toward the Earth and analyzing the reflected signals, or by directly capturing the energy that is emitted or reflected from objects on the Earth's surface. These images are then processed and analyzed to extract valuable information, which can be used for scientific research, military purposes, or commercial applications. The advantages of satellite imagery include the ability to cover vast areas in a short amount of time, providing both high-resolution imagery for detailed analysis and lower-resolution imagery for broader regional assessments.

However, despite the extensive benefits of satellite imagery, one of the main challenges is the presence of noise in the data. This noise can come from various sources, including atmospheric disturbances such as clouds, moisture, and aerosols, which can scatter or absorb electromagnetic signals. Sensor errors, such as calibration issues, pixel inconsistencies, or hardware malfunctions, can also cause noise in the imagery. Moreover, external sources of electromagnetic interference, such as radio signals or solar flares, can distort the satellite sensors' measurements, leading to inaccurate or degraded image quality.

The presence of noise in satellite imagery can significantly impact the quality of the data, making it difficult to extract accurate information. For example, in environmental monitoring, noise can obscure the detection of subtle changes in vegetation, water bodies, or land cover, making it harder to assess ecosystem health or track deforestation. Similarly, in disaster response, noise can make it difficult to identify and analyze damage in affected areas, delaying response efforts. To mitigate the effects of noise and improve the accuracy of satellite imagery, various noise reduction methods are developed. These methods may involve filtering techniques that smooth out the data or algorithms designed to detect and remove noise while preserving important features in the image. By applying noise reduction methods, the quality of satellite imagery can be enhanced, leading to more reliable and actionable insights for a wide range of applications.

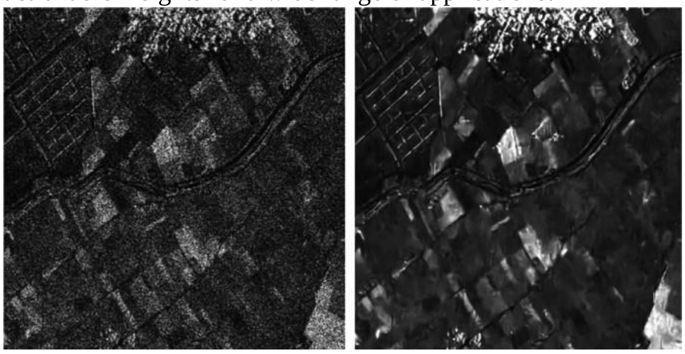

Figure 6. Noised reduction on satellite image example
Source: https://link.springer.com/chapter/10.1007/978-981-19-2358-6_62

III. IMPLEMENTATION

A. Implementation of Noise Reduction Algorithm with SVD

The application of Singular Value Decomposition (SVD) for noise reduction in satellite images involves several key steps, including matrix decomposition, selection of principal components, and image reconstruction. Below are the detailed steps:

1. Image Pre-processing
   The first step is to ensure that the image to be processed is in the appropriate format. If the input image is in RGB

format, it must be converted to grayscale. However, in the provided code, the loaded image is already in grayscale format, so this conversion step is not necessary

2. SVD Decomposition
   At this stage, the image, which is in the form of a pixel intensity matrix, undergoes decomposition using Singular Value Decomposition (SVD). This process divides the image into three matrices:
   a. U: An orthogonal matrix that represents the row structure of the image.
   b. S (Sigma): A diagonal matrix containing the singular values, which indicate the contribution of each component to the image.
   c. Vt: An orthogonal matrix that represents the column structure of the image.

   This process allows for the analysis of the contribution of each singular component to the information in the image.
3. Selection of Principal Components
   In this step, we select the most significant singular components to preserve the main information of the image and reduce noise. Larger singular values tend to represent more important image components, while smaller values often contain noise or less relevant information. In the code, this is achieved by selecting the appropriate number of singular components (determined by the rank parameter) and discarding the others. The new S matrix ($S_{reduced}$) only contains the largest singular values, with the remaining ones set to zero.
4. Image Reconstruction
   After selecting the relevant singular components, the image is reconstructed using the matrices U, $S_{reduced}$ (which contains the selected singular values), and Vt. This reconstruction process reassembles the selected components to produce a cleaner image while preserving important information. In the code, reconstruction is done using the matrix operation np.dot(U, np.dot(np.diag($S_{reduced}$), Vt)).

Below is the Python implementation that applies the noise reduction method using SVD:

```
import numpy as np
import cv2
import matplotlib.pyplot as plt
import os

def svd_noise_reduction(image, rank):

   U, S, Vt = np.linalg.svd(image,
full_matrices=False)
   # Retain only the top rank singular values
   S_reduced = np.zeros_like(S)
   S_reduced[:rank] = S[:rank]
   # Reconstruct the denoised image
   noise_reduced_image = np.dot(U,
np.dot(np.diag(S_reduced), Vt))
   # Clip values to ensure they remain in the
valid range [0, 255]
   noise_reduced_image =
np.clip(noise_reduced_image, 0, 255)
   return noise_reduced_image.astype(np.uint8)

# Load a grayscale image with noise
image_path = "./test/sample.png"
noisy_image = cv2.imread(image_path,
cv2.IMREAD_GRAYSCALE)

# Set the desired rank for noise reduction
rank = 50

noise_reduced_image =
svd_noise_reduction(noisy_image, rank)

# Display the results
plt.figure(figsize=(10, 5))

plt.subplot(1, 2, 1)
plt.title("Noisy Image")
plt.imshow(noisy_image, cmap="gray")
plt.axis("off")

plt.subplot(1, 2, 2)
plt.title("Noise-reduced Image")
plt.imshow(noise_reduced_image, cmap="gray")
plt.axis("off")

plt.tight_layout()
plt.show()

# Save the denoised image
output_path = "./test/noise_reduced_image.jpg"
cv2.imwrite(output_path, noise_reduced_image)
```

Figure 5. Implementation of noise reduction with SVD using Python

This Python code uses the Singular Value Decomposition (SVD) method to reduce noise in grayscale images. The process begins by importing the necessary modules: numpy for performing mathematical operations, cv2 for manipulating images, and pyplot from matplotlib for visualizing the image results. The main function in this code is svd_denoise, which takes an image as input and the rank parameter to determine the number of singular values to be retained during the image reconstruction.

In the svd_denoise function, SVD decomposition is first performed on the image, which has been converted into a pixel intensity matrix. This decomposition results in three matrices:

- U: An orthogonal matrix representing the row structure of the image.
- S (Sigma): A diagonal matrix containing the singular values that indicate the contribution of each component to the image.
- Vt: An orthogonal matrix representing the column structure of the image.

Next, to reduce noise, we select the largest singular values based on the provided rank parameter. Larger singular values tend to represent the main features of the image, while smaller values often contain noise. In this code, the smaller singular values are set to zero, so only the main components are retained.

After selecting the relevant singular values, the image is reconstructed using the chosen principal components. This reconstruction is performed by multiplying the U matrix, the diagonal matrix containing the selected singular values, and the

Vt matrix. The result is a reconstructed image that focuses on the main information and reduces noise.

Finally, the reconstructed image may have pixel values that fall outside the [0, 255] range. To ensure these values remain within a valid range, the np.clip function is used to clip the pixel values that exceed these limits.

B. Algorithm Testing

To demonstrate the effectiveness of the SVD algorithm in noise reduction, a test is required to evaluate the results of noise reduction performed with SVD on satellite images. Below are the results of the noise reduction test on a satellite image sample using the previously provided code:

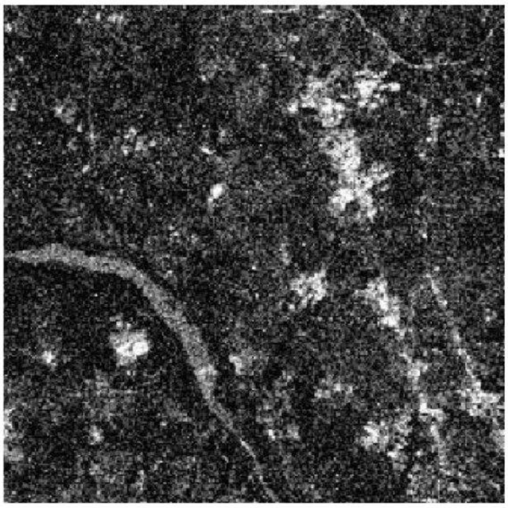

Figure 6. Satellite image with noise sample
Source: https://www.researchgate.net/figure/a-Original-satellite-image-b-Gaussian-noise-s002-image-c-Fourth-order-PDE_fig18_299649233

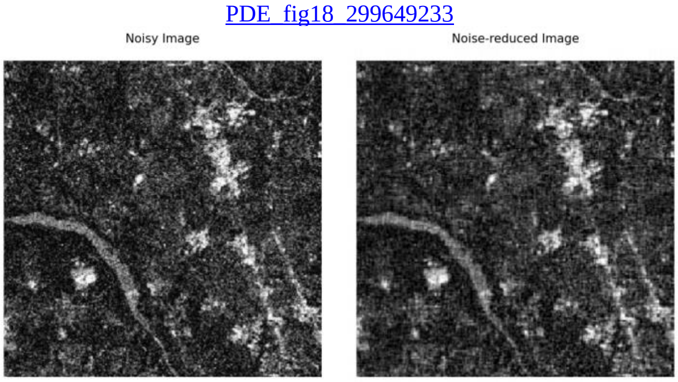

Figure 7. Noise reduction result of the sample using the given code

Based on the results above, it can be seen that the application of Singular Value Decomposition (SVD) for noise reduction on the image successfully reduced most of the noise present. However, the obtained results show a blurring effect on the image, indicating that some important components in the image representation were removed during the noise reduction process. Therefore, while this process is effective in eliminating noise components, it also reduces details that should be preserved to maintain the sharpness and quality of the original image.

This blurring effect occurs because SVD relies on removing certain singular values, which directly affects high-frequency information that often contains important details. In this context, the removal of these components contributes to the loss of image sharpness, although the noise has been successfully addressed. However, this problem becomes crucial in this context, as satellite imagery often contains fine-grained details critical for analysis.

Based on these condiserations, the method still has room for further optimization. One potential solution is to adjust the criteria for selecting components to be retained during the SVD decomposition process to maintain a balance between noise reduction and the preservation of image details. Additionally, combining the SVD method with other techniques, such as Wavelet Transform or spatial filters, could be an approach to improve the effectiveness of this method. This combined approach has the potential to produce better images by preserving important details without sacrificing the necessary noise reduction.

Overall, although SVD shows potential for noise reduction, further improvements are needed, either through parameter adjustments or by combination with other methods to achieve more optimal results.

IV. Conclusion

Based on the results of the trials obtained, it can be concluded that although the SVD method successfully reduces noise in the image, the results are still unsatisfactory because the image becomes slightly blurred. This happens because some components in the image representation with small singular values are removed during the noise reduction process, leading to the excessive removal of components. Nevertheless, the SVD method can be further optimized by adjusting the criteria for the components that are retained during the noise reduction process. Additionally, to improve the results, SVD can be combined with other techniques such as wavelet transform or spatial filters, which allow for the recovery of image details while maintaining effective noise reduction. This combination can provide better results, preserving image quality while reducing noise more efficiently.

V. Suggestion

The author suggests that future researchers who wish to study the same topic focus on improving the effectiveness of the SVD method in noise reduction by making various modifications, such as adjusting the criteria for selecting the components to be retained, ensuring that important components are preserved and the image quality does not decrease significantly. Additionally, combining SVD with other techniques such as wavelet transform or spatial filters could help recover lost image details without reducing the effectiveness of noise reduction. Exploring other noise reduction algorithms, such as deep learning or statistical model-based methods, could also be an alternative for achieving better results.

VI. Apendix

The GitHub repository for this research can be accessed at https://github.com/JethroJNS/Algeo-Noise-Reduction-Using-SVD.git and the explanatory video for this research can be accessed at https://youtu.be/8Y5YyhbF4ug

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STATEMENT

Hereby, I declare that this paper I have written is my own work, not an adaptation or translation of someone else's paper, and not a product of plagiarism.

Bandung, 26th December 2024

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